

Optimal behavior of viscoelastic flow at resonant frequencies

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The global entropy generation rate in the zero-mean oscillatory flow of a Maxwell fluid in a pipe is analyzed with the aim of determining its behavior at resonant flow conditions. This quantity is calculated explicitly using the analytic expression for the velocity field and assuming isothermal conditions. The global entropy generation rate shows well-defined peaks at the resonant frequencies where the flow displays maximum velocities. It was found that resonant frequencies can be considered optimal in the sense that they maximize the power transmitted to the pulsating flow at the expense of maximum dissipation.

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I. INTRODUCTION

There are several interesting phenomena where the existence of an oscillatory flow leads to the improvement of a transport process [1]. For instance, the axial dispersion of contaminants within laminar oscillatory flows in capillary tubes is considerably larger than that obtained by pure molecular diffusion in the absence of flow [2,3]. Likewise, Kurzweg [4,5] found that in a zero-mean oscillatory flow of a Newtonian fluid in a duct, the effective thermal diffusivity reaches a maximum for a given oscillation frequency. This leads to an enhanced longitudinal heat transfer which involves no net convective mass transfer. In turn, it has been found [6–8] that the dynamic permeability of a viscoelastic fluid flowing in a tube can be substantially enhanced at specific resonant oscillation frequencies. Under certain conditions, an enhanced flow rate can be achieved. These phenomena may find important applications in areas of technological interest such as nuclear reactors, combustion processes, and oil recovery [9,10], as well as for the understanding of physiological flows such as those present in respiratory and circulatory systems [11,12].

In this paper, we are interested in the relation between the irreversible behavior of an oscillating flow and the optimal characteristics of the enhanced transport. In recent years, a variety of systems have been analyzed and optimized using the entropy generation minimization method [13–16]. This method has become a useful tool for evaluating the intrinsic irreversibilities associated with a given process or device. By determining the conditions under which the entropy generation rate is minimized, the operating conditions can be optimized by reducing the dissipation to a minimum consistent with the physical constraints imposed on the system. In fluid flow systems, friction is one of the main mechanisms responsible for entropy generation; therefore, we must invest useful work to push the fluid through the pipe against the irreversible viscous dissipation. In this work, the entropy generation

rate is used to evaluate the intrinsic irreversibilities associated with an oscillatory viscoelastic flow. Some interesting applications of thermodynamic optimization have been proposed by Bejan in the context of pulsating flows [11,12]. In particular, he has shown that in the respiratory system, the minimization of the mechanical power requirements by the thorax muscles during the inhaling and exhaling cycle corresponds to the longest inhaling and exhaling strokes possible, while in ejaculation, the maximization of the mechanical power transmitted to the ejected seminal fluid explains the existence of an optimal bursting time interval. It has to be pointed out that these works consider only the viscous dissipative behavior of fluids. However, most of biological fluids present a viscoelastic nature and improved calculations must also reflect their elastic behavior. In fact, del Río *et al.* [7] speculated that the human heart beats at the *optimum* pumping frequency to produce a maximum flow through arteries and veins according to the viscoelastic properties of the blood. Recently, this resonant behavior was experimentally observed in a study of the dynamic response of a Maxwellian fluid [17], where the enhancement at the frequencies predicted by the theory was proved. In turn, Tsiklauri and Beresnev [18,19] included the effect of longitudinally oscillating tube walls and obtained the analogous enhanced behavior. All these results have motivated exploration of the consequences of the enhancement of the dynamic response of an oscillating viscoelastic fluid under different conditions [20–22]. At this point, the question whether this *optimum* pumping behavior is also optimum or efficient from a thermodynamical point of view can be formulated. This problem can be addressed through the analysis of the entropy generation rate [13,14]. In this paper, we have focused our attention on the analysis of the relationship between maximum permeability (and, therefore, maximum velocity) of a zero-mean oscillatory flow of a viscoelastic fluid in a rigid cylindrical tube and the entropy generation rate that characterizes the process. From the analytic expression for the velocity field, shear stresses are determined, and the local and global entropy generation rate as a function of the oscillation frequency are calculated. In this work, irreversibilities due to heat flow phenomena are not considered.

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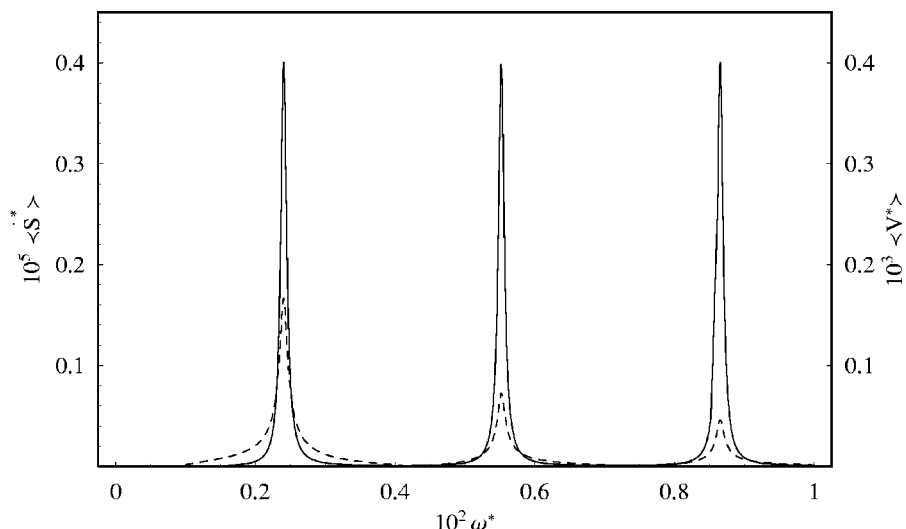


FIG. 1. The amplitudes of the dimensionless velocity $\langle V^* \rangle$ (dashed line) and global entropy generation rate $\langle \dot{S}^* \rangle$ (solid line) as a function of the dimensionless frequency ω^* with $\alpha=0.01$.

II. THEORETICAL MODEL

We consider the flow of a Maxwell fluid in a rigid cylindrical tube of radius a under an oscillatory pressure gradient applied in the longitudinal x direction. This problem was solved analytically by del Río *et al.* [7] in the linear regime, and the corresponding velocity field $V(r, t)$ reads

$$V(r, t) = -\frac{1 + i\omega t_m}{\beta^2 \eta} \left(1 - \frac{J_0(\beta r)}{J_0(\beta a)} \right) \frac{dP}{dx}, \quad (1)$$

where the no-slip condition has been imposed at the wall of the cylinder, $V(a) = 0$. Here $\beta = \sqrt{(\rho/\eta t_m)[(t_m \omega)^2 - i\omega t_m]}$, η and ρ are the dynamic viscosity and mass density of the fluid, t_m is the relaxation time for the Maxwell fluid, J_0 is the cylindrical Bessel function of zeroth order, and dP/dx is the general expression of the time-dependent pressure gradient. All physical properties of the fluid are considered constant. In order to obtain analytical results, in this work we chose a harmonic pressure gradient given by the real part of the expression $P_x e^{-i\omega t}$, where P_x is the constant amplitude of the pressure gradient and ω is the angular frequency. With this assumption, the dimensionless expression for the velocity field is

$$V^*(r^*, t^*) = -\frac{1 + i\omega^*}{\alpha \varpi} \left(1 - \frac{J_0(\sqrt{\alpha \varpi} r^*)}{J_0(\sqrt{\alpha \varpi})} \right) e^{-it^*}, \quad (2)$$

where V^* , ω^* , r^* , and t^* have been normalized by $V_0 = (a^2/\eta)P_x$, $1/t_m$, a , and $1/\omega$, respectively. Here, $\varpi = (\omega t_m)^2 - i\omega t_m$ while $\alpha = a^2 \rho / \eta t_m$ is the Deborah number.

A. Entropy generation rate

We now proceed to calculate the entropy generation rate. Since the fluid is assumed to be a simple substance, mass diffusion phenomena are disregarded. In addition, we consider that the main source of entropy generation is given by frictional effects. However, it is assumed that the rise in temperature in the fluid and walls due to this dissipative effect is negligible so that temperature remains approximately constant and irreversibilities due to heat transfer are not taken

into account. Under these approximations, the dimensionless local entropy generation rate \dot{S}^* that characterizes the irreversible behavior of the system is given by

$$\dot{S}^*(r^*, t^*) = \frac{1}{T^*} \left(\frac{\partial V^*}{\partial r^*} \right)^2, \quad (3)$$

where \dot{S}^* and the dimensionless temperature of the fluid T^* are normalized by $V_0^2 \eta / T_0 a^2$ and T_0 , respectively, T_0 being the mean dimensional fluid temperature. Notice that V^* and \dot{S}^* always are in phase, the temporal variation being $\cos(t^*)$ and $\cos^2(t^*)$, respectively. In order to obtain the entropy generation rate per unit length in the axial direction, $\langle \dot{S}^* \rangle$, \dot{S}^* is integrated over the tube cross section. Thus, $\langle \dot{S}^* \rangle$ is a function of only t^* , ω^* , and α . The corresponding averaged velocity over the tube cross section is

$$\langle V^* \rangle = \frac{2\pi}{A} \int_0^1 V^*(r^*, t^*) r^* dr^*, \quad (4)$$

where A is the cross section area. We can now use Eqs. (3) and (4) to characterize the resonant behavior of the system.

III. RESULTS

In Fig. 1, the amplitudes of the averaged velocity and the global entropy generation rate are shown as a function of the dimensionless frequency for a Deborah number $\alpha=0.01$. For comparison purposes, we have used the same value of α as in the paper by del Río *et al.* [7]. It corresponds to a fluid with a relaxation time of the order of seconds, a mass density and viscosity of the same order of water, and a tube radius of the order of centimeters. With this value, viscoelastic behavior is well established. In fact, the physical properties of cetylpyridinium chloride and sodium salicylate solution (CPyCl/NaSal, 60/100) [23,24] give an α value close to 0.01. For simplicity, in all calculations presented here, the dimensionless temperature was taken as $T^*=1$. Notice that the maximum values of $\langle \dot{S}^* \rangle$ are found at the resonant frequencies where $\langle V^* \rangle$ is also maximum. This has important

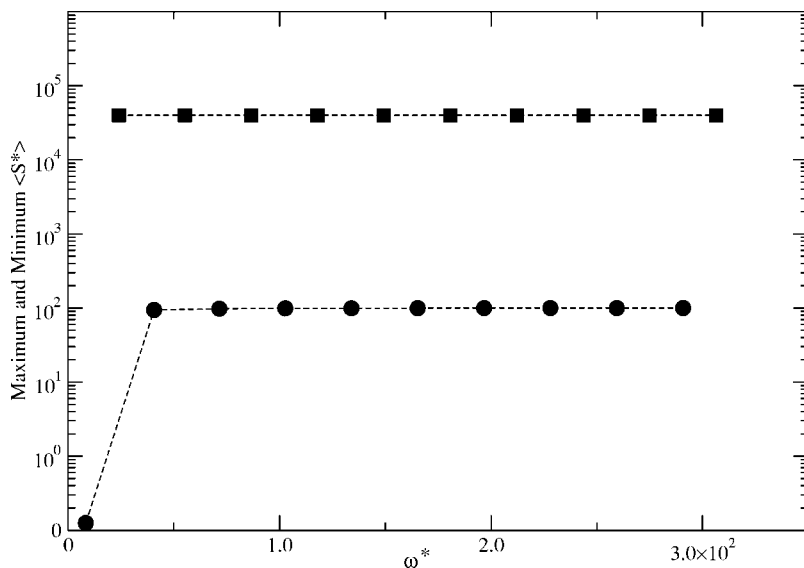


FIG. 2. Maximum (square) and minimum (dot) values of $\langle \dot{S}^* \rangle$ at different resonant frequencies. The dashed lines show only the trend behavior.

implications in terms of the useful work that is invested to move the fluid through the pipe: maximum velocity is obtained at the expense of maximum dissipation. On the other hand, from the relationship between work W and velocity v , namely, $dW/dt = PAv$, it is clear that for a given pressure P and cross-sectional area A , maximum fluid velocity leads also to maximum power. Therefore, it follows that resonant frequencies can be considered optimal in the sense that they maximize the power transmitted to the fluid through the pulsating flow.

An interpretation of this result in terms of Darcy's law can also be given. The phenomenological law for a frequency-dependent mean flux (or average velocity) can be expressed as $\mathbf{J} = \langle \mathbf{V}^* \rangle = -K(\omega^*) \nabla P$, where $K(\omega^*)$ is the dynamic permeability [7]. Therefore, expressing the global entropy generation rate as the product of fluxes and generalized forces [25], we get

$$\langle \dot{S}^* \rangle = -\frac{1}{T} \mathbf{J} \cdot \nabla P = \frac{K(\omega^*)}{T} |\nabla P|^2 \quad (5)$$

where in order to satisfy the condition $\langle \dot{S}^* \rangle \geq 0$, the dynamic permeability must be a positive definite quantity. From Eq. (5), it is then clear that maximum values of $\langle \dot{S}^* \rangle$ will be obtained at those frequencies at which the $K(\omega^*)$ is maximized. But from Darcy's law these are precisely the frequencies that lead to maximum mean flux or average velocity.

It is also interesting to observe in Fig. 1 that while maximum values of $\langle \mathbf{V}^* \rangle$ decrease as higher resonant frequencies are reached, maximum values of $\langle \dot{S}^* \rangle$ remain almost constant. This is more clearly shown in Fig. 2 where maxima and minima of the global entropy generation rate are presented as a function of the frequency. This result indicates the importance of the first resonant frequency where the higher mean velocity is obtained. The irreversibilities associated with the production of the first peak velocity are approximately the same as those involved in the production of the remaining peaks although maximum velocity values decrease the higher the frequency. A drastic rise in the minima

is observed from zero frequency to the first minimum, but from that value the remaining local minima stay almost constant and, in fact, they reach a limit value as $\omega^* \rightarrow \infty$. It is important to emphasize the fact that the lowest minimum of the global entropy generation rate corresponds to a stationary state, i.e., to the zero frequency. This result is in agreement with Prigogine's theorem, which states a minimum entropy generation for stationary states provided that the Onsager coefficients are constant [25].

IV. CONCLUSIONS

In this paper, we have used the global entropy generation rate to analyze a zero-mean oscillatory flow of a Maxwell fluid at resonant conditions. It was found that the global entropy generation rate is maximized at the same frequencies at which the flow displays a resonant behavior. Under these conditions the average velocities are maximum and the power transmitted to the fluid through the pulsating flow is also maximum. Therefore, it is from the maximization of power that pumping at resonant frequencies can be considered optimal. However, from a thermodynamic point of view, maximum average velocities are reached through the maximization of flow irreversibilities. Given the viscoelastic nature of most biological fluids, this may help in the understanding of some pulsating physiological processes [7,11,12].

It was observed that global entropy generation rate remains the same at different resonant frequencies although the maximum values of velocity decrease at higher frequencies. On the other hand, the existence of a lowest minimum value of $\langle \dot{S}^* \rangle$ is in agreement with Prigogine's theorem of minimum entropy production for stationary states.

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